An Introduction to Game Theory

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Outline

- 1. Background Information
- 2. Two Person Zero-Sum Games
- 3. Two Person Non-Zero-Sum Games
- 4. N-Player Games

History

- 1928: John von Neumann published the minimax theorem
- 1944: *Theory of Games and Economic Behavior* by von Neumann
- 1950: Prisoner's dilemma formulated by Albert Tucker
- 1950: John Nash introduced the "Nash Equilibrium"
- 1970: John Maynard Smith applies Game Theory to Biology

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Problem Description

A Game Theory problem consists of the following:

- The number of players (often 2 players)
  - All players are assumed to play intelligently and accurately
  - Players may or may not be able to communicate
- A payoff matrix
- The number of times the game will be played

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Problem Example

Companies A and B plan to buy 40 and 24 new forklifts respectively. Two salesmen, Frank and Joe, are competing to win the sales contracts from these companies. On the final day of negotiations, the salesmen can visit one but not both of the companies to pitch their product one last time. If both salesmen visit the same company they will split the order, if they are the only one to visit the company they will secure the entire order, and if neither of them shows up at a company, all the sales will go to Joe since his company has been in business longer.

Problem Example

Company A: 40 forklifts
Company B: 24 forklifts

Joe

Frank		A	B
	A	(20,44)	(40,24)
	B	(24,40)	(12,52)

Problem Example

Company A: 40 forklifts
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Joe

Frank

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A	20	40
B	24	12

Solution Description

A solution to a Game Theory problem, often called a strategy, is a complete plan of action that maximizes/minimizes the players payoff over the duration of the problem.

- A **pure strategy** consists of making the same choice each time the game is played in the problem.
- A **mixed strategy** consists of varied choices for each time the game is played. These choices may be made based on a random device or on the previous moves of other players.

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Solution Example

A pure strategy for Frank is to visit A every time since he will at least sell 20 forklifts.
A mixed strategy for Frank is to visit A half of the time and B half of the time.

Joe

Frank

	A	B
A	20	40
B	24	12

Zero-Sum Games

Zero-Sum Games

Definition

Definition (Zero-Sum Game)

A **zero-sum** game is one in which the net payoff for all players is zero. Thus, in a two player game, one player's gain is the other player's loss.

Examples of zero-sum games include:

- board games like Othello
- people dividing a cake between themselves
- prides of lions establishing their territory

Zero-Sum Example

Player 2

Player 1

	D	E
A	(15,10)	(10,15)
B	(6,19)	(15,10)
C	(5,20)	(20,5)

Zero-Sum Example

Democrats

Republicans

	Favor A	Favor B	Dodge
Favor A	45%	50%	40%
Favor B	60%	55%	50%
Dodge	45%	55%	40%

- The Republicans always do better favoring issue B.
- The Democrats always do better dodging the issue.
- The solution to this game is for the Republicans to favor issue B and the Democrats to dodge the issue. The resulting payoff will be 50% of the vote for each party.

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- The Democrats should not dodge the issue.
- The Republicans should then favor issue B.
- The solution to this game is for the Republicans to favor issue B and the Democrats to favor issue A. The resulting payoff will be 45% of the vote for the Republicans and 55% for the Democrats.

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Definition

Definition (Dominated Strategy)

A **dominated strategy** for a game is one in which another strategy provides a higher payoff regardless of the other players actions.

Definition (Equilibrium)

A pair of strategies are in **equilibrium** if no player can increase their payoff by changing their strategy unilaterally. The pair of strategies is itself called an **equilibrium** point.

Definition (Value of a game)

The **value of a game** is the payoff associated with an equilibrium point.

Minimax

Theorem (Minimax)

For every two-person, zero-sum game with finite strategies, there exists a value V, called the value of the game, and a mixed strategy for each player, such that

(a) *Given player 2's strategy, the best payoff possible for player 1 is V, and*

(b) *Given player 1's strategy, the best payoff possible for player 2 is -V.*

Finding Solutions

Calculate the minimax and the maximin of the game

1. If they are equal, you have found a pure strategy for both players and the payoff for player 1 is gauranteed to be at least the value of the game.
2. If they are not equal, you can solve the problem using Linear Programming or Calculus, and you will obtain the value of the game on average.

Linear Programming Solution

Consider a game with payoff entries M_{ij} that has a value of V, and player 1 makes choice i with probability p_i. We want to choose p_i in proportion to the amount of the value of the game that we get when choosing that p_i. Thus,

$$p_i = u_i \cdot V.$$

Since p_i's are probabilities we know that they sum to 1.

$$\begin{aligned} \sum_i u_i \cdot V &= 1 \\ V \cdot \sum_i u_i &= 1 \end{aligned}$$

Since we want V to be as large as possible, our objective function is

$$\min \sum_i u_i$$

Linear Programming Solution

For any pure strategy that player 2 adopts we want our payoff to be at least the value of the game. Thus,

$$\sum_i M_{ij} p_i \geq V \;\; \forall j.$$

Rearranging the above slightly and including the non-negativity of u_i gives the constaints for our problem.

$$\begin{aligned} \sum_i M_{ij} u_i &\geq 1 \;\; \forall j \\ u_i &\geq 0 \;\; \forall i \end{aligned}$$

Linear Programming Solution

The complete program is given as follows

$$\min \sum_i u_i$$

s.t.

$$\begin{aligned} \sum_i M_{ij} u_i &\geq 1 \ \forall j \\ u_i &\geq 0 \ \forall i \end{aligned}$$

Once we solve this we can determine the value of the game since

$$V = \frac{1}{\sum_i u_i}.$$

Calculus Solution

Consider a game with 2 choices for each player. The game has payoff entries M_{ij}, and player 1 makes choice 1 with probability p and player 2 makes choice 1 with probability q.
Player 1 seeks to find p such that his average payoff is the same regardless of which q player 2 is using. Thus,

$$f(p,q) = q(pM_{11} + (1-p)M_{21}) + (1-q)(pM_{12} + (1-p)M_{22})$$

is his average payoff as a function of p and q. Using elementary calculus, we can find the optimal p by solving for p such that $\frac{\partial f}{\partial q} = 0$.

$$p = \frac{M_{22} - M_{21}}{(M_{22} - M_{21}) + (M_{11} - M_{12})}$$

Zero-Sum Example

Player 2

Player 1

	D	E
A	15	10
B	6	15
C	5	20

Playing B is dominated by playing A 1/5 of the time and B the other 4/5 of the time.

Zero-Sum Example

Player 2

Player 1

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A	15	10
B	6	15
C	5	20

Playing B is dominated by playing A $1/5$ of the time and B the other $4/5$ of the time.

Zero-Sum Example

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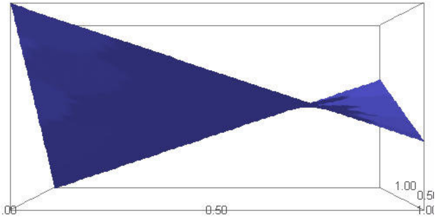
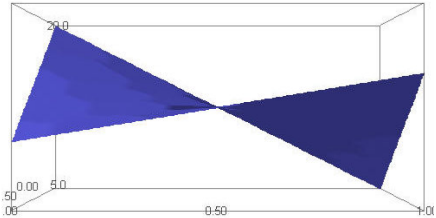
Zero-Sum Example Solution

p-axis:



q-axis:



Zero-Sum Example Solution

Using the formula from the calculus derivation we obtain

$$\begin{aligned} p &= \frac{20-15}{(20-15)+(15-10)} = \frac{3}{4} \\ q &= \frac{5-15}{(5-15)+(10-20)} = \frac{1}{2} \end{aligned}$$

The value of the game is $f\left(\frac{3}{4}, \frac{1}{2}\right) = 12.5$.

Non-Zero-Sum Games

Non-Zero-Sum Games

Definition

Definition (Non-Zero-Sum Game)

A **non-zero-sum** game is one in which the net payoff for all players is not zero.

Examples of non-zero-sum games include:

- prisoner's dilemma
- two countries engaging in trade
- gas stations competing on the same corner

Non-Zero-Sum Example

Player 2

Player 1

	C	D
A	(1,3)	(5,1)
B	(0,-90)	(0,-100)

Considerations

- How many times will the game be played?
- Can players communicate?
- Can players make agreements in advance and are they binding?
- Are the players cooperating or competing?

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Analysis

- Strategies can be dominated just like in zero-sum games.
- Minimizing the other player's payoff will not necessarily maximize your payoff.
- Ultimately, there may not be one best solution so various heuristics have arisen to try to solve these problems.

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Communication Example

Police

Gang		Negotiate	Do Not Negotiate

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Gang	Harm Hostages	A	B
	Do not Harm Hostages	C	D

- The gang probably won't do A and would prefer C most, then D, and least of all B.
- The police want D the most, then C, and lastly B.

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Prisoner's Dilemma

Suspect 2

Suspect 1

	Confess	Don't Confess
Confess	(5,5)	(0,20)
Don't Confess	(20,0)	(1,1)

- Each suspect appears to do better by confessing.
- Their individual best is not their collective best.

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Prisoner's Dilemma

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- If the game is played once, both players will confess.
- If the game is played a fixed number of times, both players will confess.
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Hawk-Dove

- Two types of birds, hawks and doves, meet and compete for food worth 10 utils.
- If two hawks fight over the food, the winner gets 10 utils and the loser gets -20 utils.
- If two doves fight over the food, the winner gets 10 utils and both get -3 utils.
- Doves will always back away from a fight with a hawk

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John Maynard Smith proposed that a population, I, is evolutionary stable if for any alternative population, J, one of the following hold:

- $E(I, I) > E(J, I)$
- $E(I, I) = E(J, I)$ and $E(I, J) > E(J, J)$

For the Hawk-Dove scenario, a mixed population having $\frac{8}{13}$ hawks and $\frac{5}{13}$ doves is evolutionary stable.

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N-Player Games

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Pirate Example

A band of pirates has captured and plundered a Spanish galleon. Part of the treasure is a chest containing 100 gold coins. They decide to divide the treasure as follows:

- Each pirate, in order of age, will propose a distribution of the coins. All the pirates will vote whether to accept his plan or not.
- If the plan is approved, or there is a tie, the plan will be adopted.
- If the plan is not approved, the pirate who proposed the plan must walk the plank, and the next pirate gives his plan.

The pirates decide whether to accept a plan based on the following priorities.

1. Each pirate wants to survive.
2. Each pirate wants to maximize the amount of gold he gets.
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Considerations

- Many of the same considerations from non-zero-sum games hold.
- Players may form coalitions and vote together on issues.
- Bargaining power must be accounted for.

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Pirate Solution

To find the solution we work the problem backwards.

- 2 pirates, A:100, B:0
- 3 pirates, A:99, B:0, C:1
- 4 pirates, A:99, B:0, C:1, D:0
- 5 pirates, A:98, B:0, C:1, D:0, E:1

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- 3 pirates, A:99, B:0, C:1
- 4 pirates, A:99, B:0, C:1, D:0
- 5 pirates, A:98, B:0, C:1, D:0, E:1

Pirate Solution

To find the solution we work the problem backwards.

- 2 pirates, A:100, B:0
- 3 pirates, A:99, B:0, C:1
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Job Example

An accountant, an engineer, and a lawyer are offered jobs at a company. If they join together as a team they are offered bonuses as shown below.

Team	Bonus
A and E	20
A and L	24
E and L	16
All three	28

Job Example

Team	Bonus
A and E	20
A and L	24
E and L	16
All three	28

- Which team should join?
- How should they divide the bonus?

References

- *Game Theory: A Nontechnical Introduction* by Morton Davis
- www.wikipedia.org